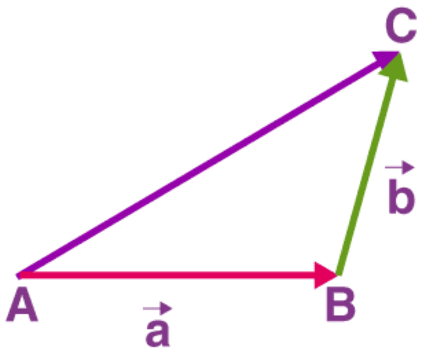
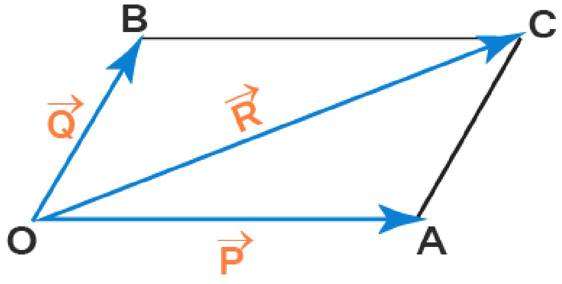
Vectors and Coordinates #2

YORK COLLEGE & UNIVERSITY CENTRE

Adding Vectors

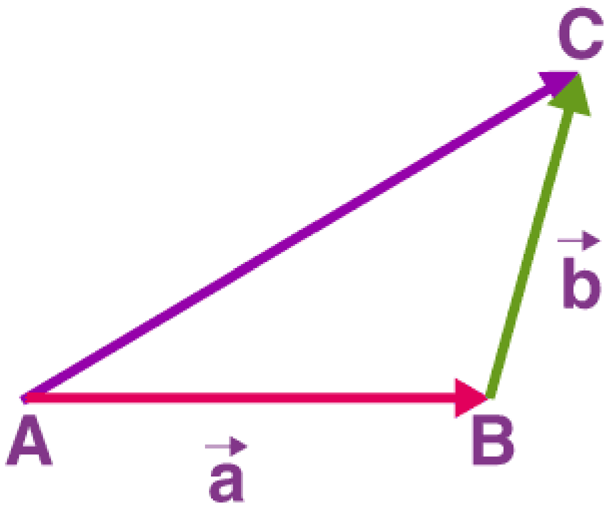
- There are several methods to add two vectors together
- There are two main graphical methods:
  - Tip-to-Tail Method
  - Parallelogram Method
- And one main mathematic method:
  - Component addition
- **Note graphical methods only work when drawing to scale**





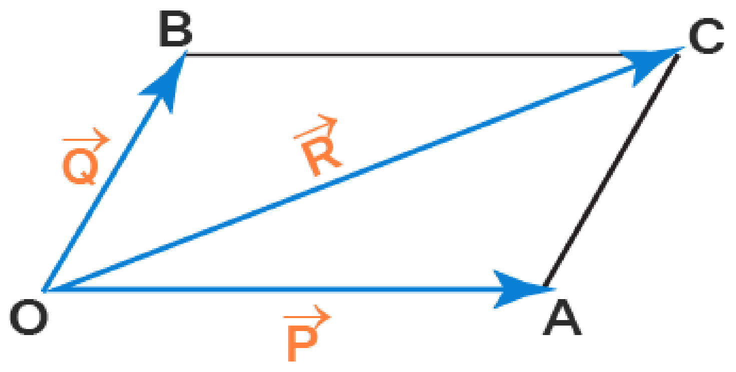
Tip to Tail Method

- In tip to tail method you draw one vector on a graph
- Then draw the next vector extending from the tail of the original
- We can repeat this for however many vectors you want to add
- Then we can measure the final line and work out the angle using a ruler and a protractor



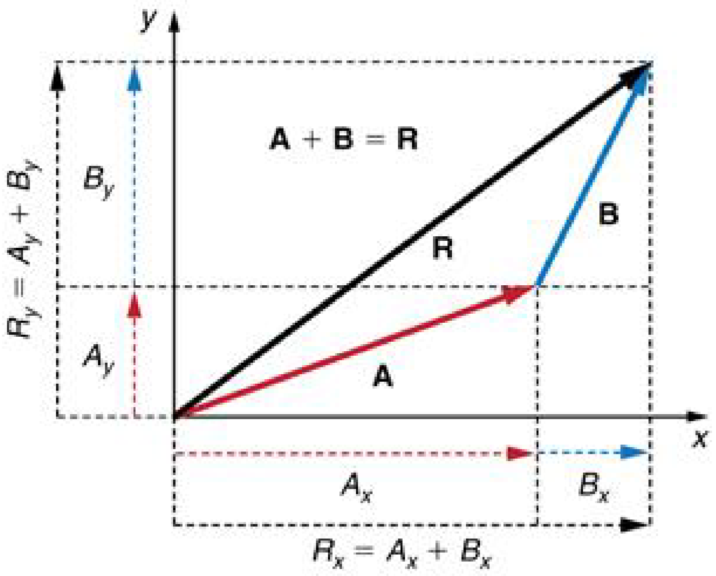
Parallelogram Method

- In the parallelogram method we draw out the vectors we are adding starting at the 0 point
- We then take these two lines to draw a parallelogram
- We then work out the distance from the 0 point to the far end of that parallelogram
- We can also work out the angle for this line for the resultant vector



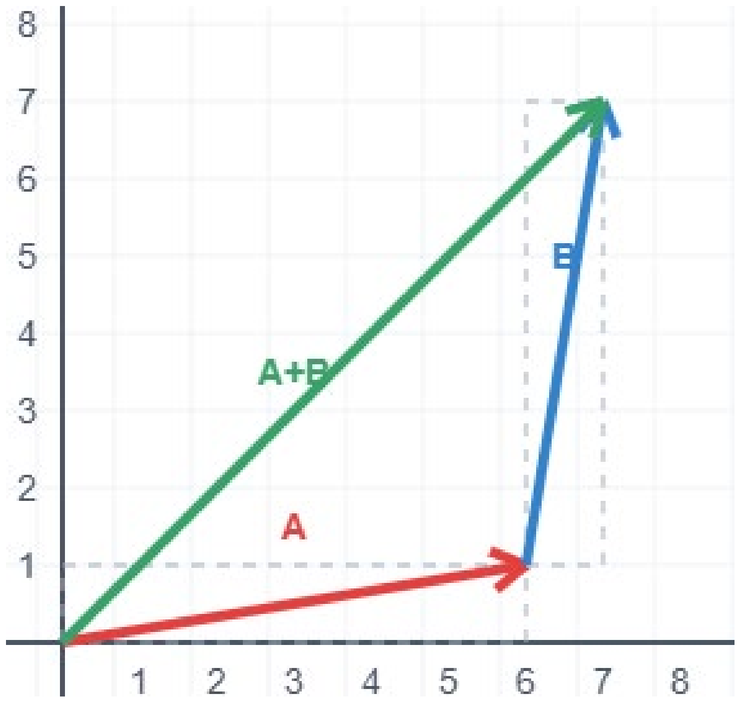
Component Addition

- For component addition we split both vectors into X and Y
- We then add both Xs and both Ys to give us a final resultant cartesian vector
- We can then do cartesian to polar conversion to find a resultant vector



Component addition example

- We can look at the graph on this page, we know that A is (6,1)
- We can also determine that B is (1,6)
- So, if we add our x components: 6+1 = 7
- Then if we add our y components: 1+6 = 7
- Then we get the final vector (7,7)



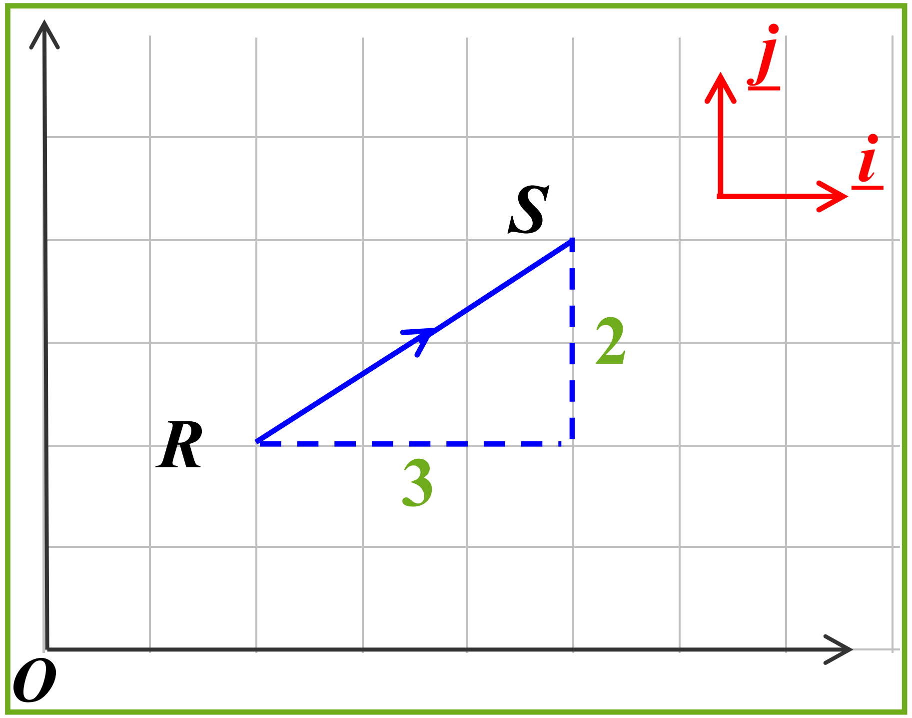
i and j

- If we don't want to constantly draw out our vectors or clarify them with $\overrightarrow{AB}$ we can write out a vector with i and j

- i is any movement in the x axis

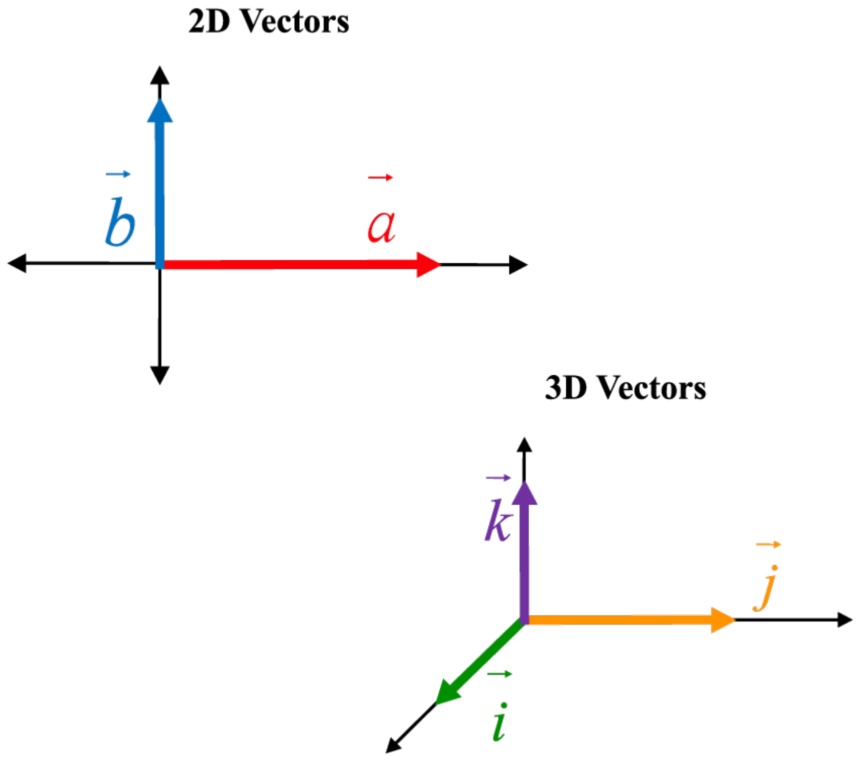
- j is any movement in the y axis

- So we can write $v = 3i + 2j$



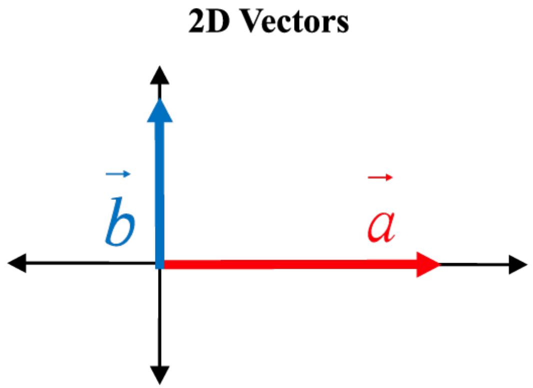
What are 3D vectors

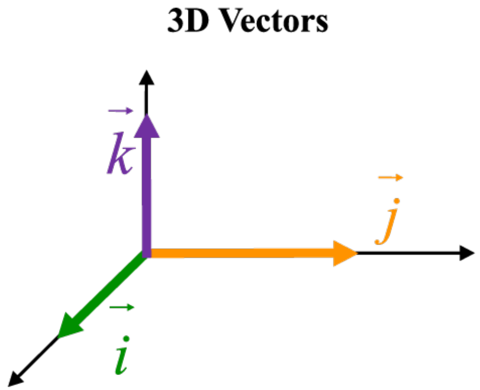
- 3D vectors are just vectors with 3 values rather than 2
- They're used for real-world 3D spaces
- They can be used to visualise position as well as velocity
- They're used for analysing structures, fluid flows, stress, and movement in machines or vehicles.



What are 3D vectors

- 2D vectors have x and y (a and b)
- 3D vectors have x, y and z (i, j, k)
- We can write 2D vectors as (x + y)
- We can do the same with 3D vectors (x + y + z)





Forms of 3D Vectors

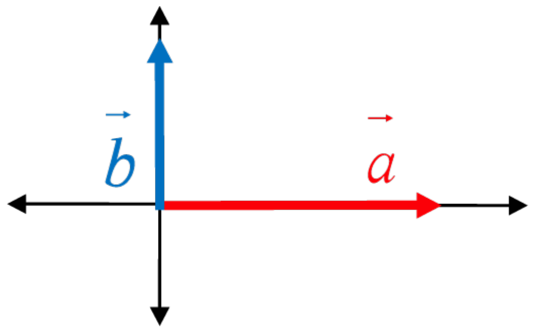
- There are two different ways of writing 3D vectors

- These are either a column or unit

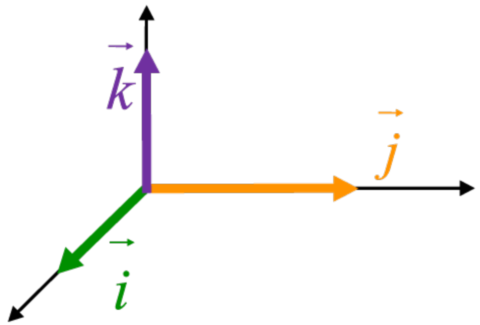
- Column = $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

- Unit = $xi + yj + zk$

2D Vectors

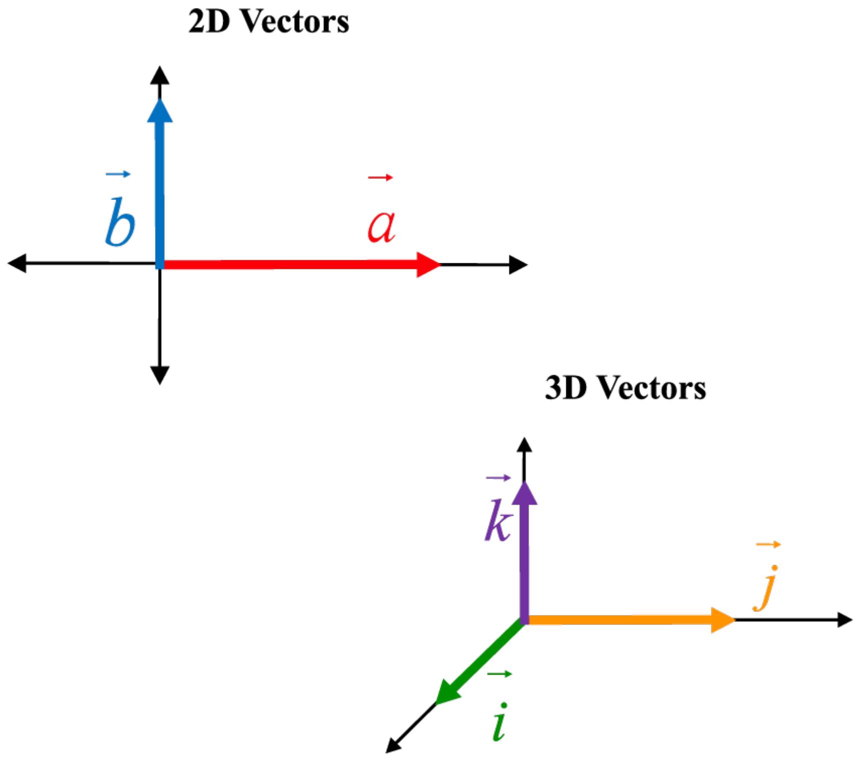


3D Vectors



Example of 3D vector types

- Column = $\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} = 3\mathrm{i} - 2\mathrm{j} + 4\mathrm{k}$



Calculating the magnitude

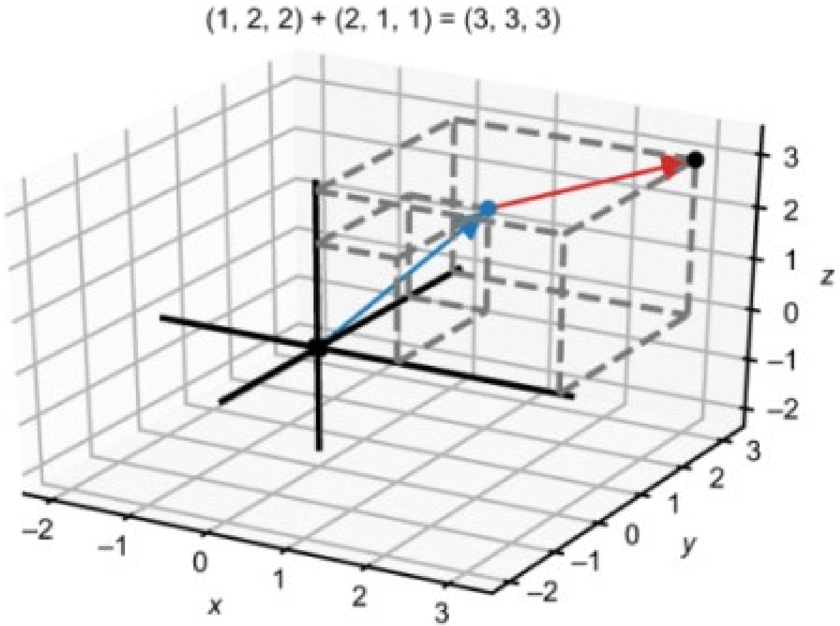
- To calculate the magnitude of a 3D vector we just use Pythagorean theorem
- $|a| = \sqrt{x^2 + y^2 + z^2}$
- Note we can forget the minus in front of any of the values as it is squared anyway

MAGNITUDE

$$|\mathbf{a}| = |x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \left|\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right| = \sqrt{x^2 + y^2 + z^2}$$

Calculating a resultant

- Calculating the resultant for multiple 3D vectors is easy
- All we do is add the different components
- So $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{pmatrix}$

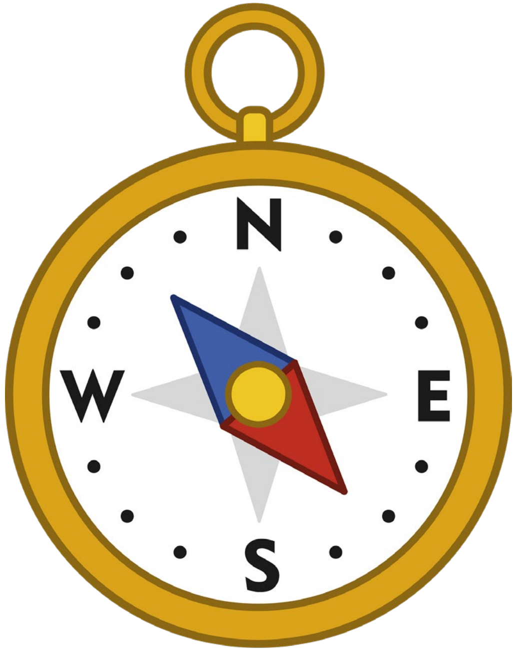


Normalising 3D Vectors

- We normalise 3D vectors to separate magnitude and direction, we do this by making magnitude 1

- This is helpful as it allows us to apply any magnitude to that vector

- Simple analogy:
- Think of a compass needle.
- The compass shows you direction (north, east, etc.).
- It doesn’t matter if the needle is long or short — the direction is what matters.
- Normalized vectors are like compass needles: size = 1, direction = important.



Steps to normalise a 3D vector

- To find the normalised vector, we follow these steps:

1) Calculate the magnitude of the vector ($|V_1|$)

2) Put the magnitude and the vector into this equation:

$$U_1 = \left(\frac{x}{|V_1|}, \frac{y}{|V_1|}, \frac{z}{|V_1|}\right)$$

What to do with a normalised 3D Vector

- If you are given a magnitude in the question usually the question will then expect you to multiply the magnitude by the normalised vector

- So you'll get:

$$U_1 = N * \left(\frac{x}{|V_1|}, \frac{y}{|V_1|}, \frac{z}{|V_1|}\right)$$

Where N is the new magnitude, we are putting in

Example Question

- We have a force acting in direction $V_1 = \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix}$ with a magnitude of 12N we need to find the normalised vector then work out the vector for that magnitude

- Step 1:

  Let's work out the magnitude of the vector ($|V_1|$)

  $|V_1| = \sqrt{4^2 + 6^2 + 2^2} = 7.483314774$

Example Question

- Step 2:

  Let's calculate the new normalised vector

  $$U_1 = \left(\frac{4}{7.483}, \frac{6}{7.483}, \frac{-2}{7.483}\right) = (0.535, 0.802, -0.267)$$

- Step 3:

  Now let's add in our new magnitude

  $$12 * (0.535, 0.802, -0.267) = (6.414, 9.621, -3.207)$$